7. FIVE DIMENSIONAL WAVE-EQUATION FOR HYDROGEN ATOM

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Introducing a new co-ordinate s on par with the usual x, y and z co-ordinates, Banerjee (1957) has set up the five dimensional wave equation given by

$$[c^2(p_x{}^2+p_y{}^2+p_z{}^2+p_s{}^2)+m_o{}^2c^4]\psi = [W-V(r)]^2\psi \qquad \dots \quad (1)$$

Taking $V = (-Ze^2/r)$ as the hydrogen atom potential, he has solved this equation in four dimensional polar co-ordinates to obtain the solution

$$\psi(r, \theta, \phi, \chi) = DR_{n_r,l}{}^{(r)}P_l^m(\cos\theta)e^{im\varphi}\sin{}^l\chi \qquad \dots \quad (2)$$

The eigenvalues of W, corresponding to the above eigen-functions, have been shown to be those given by Sommerfeld's (1916) formula.

The above wave-equation possesses the usual disadvantages associated with second order temporal differentiation contained therein. Further, the degeneracy is given by $(2l+1)$ which is unlike that of the Dirac levels. However, the equation for χ given as

$$\left[\frac{\partial}{\partial\chi}(\sin{}^2\chi\ \frac{\partial}{\partial\chi}+\left\{K\sin^2\chi-l(l+1)\right\}\right]U(\chi) = 0 \qquad \dots \quad (3)$$

can be shown to possess polynomial solutions of the form

$$U(\chi) = \sum_{\mu=o}^{l'} a_\mu \sin^{l+2\mu}\chi$$

which leads to the degeneracy $\{(l+2l'+1)(l+2l'+2)/2\}$ which is again unlike that obtained for Dirac levels.

Banerjee has given the radial equation as

$$\left[\frac{d^2}{dr^2}+\frac{3}{r}\frac{d}{dr}+\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)\right]R(r) = 0 \qquad \dots \quad (4)$$

where $\quad A = \dfrac{w^2-m_o{}^2c^4}{\hbar^2c^2}$; $B = \dfrac{WZe^2}{\hbar^2c^2}$ and $c = [a^2Z^2-l(l+2)]$

The only difference introduced by using the above polynomial solutions for χ would be to replace l by $(l+2l')$. Substituting $R(r) = e^{-\rho/2}v(\rho]$ with $\rho = (2r/r_0)$ where $r_0 = \frac{1}{\sqrt{-A}}$, one obtains

$$v''+v'\left(\frac{3}{\rho}-1\right)+\left[\left(\frac{B}{\sqrt{-A}}-\frac{3}{2}\right)\frac{1}{\rho}+\frac{c}{\rho^2}\right]v=0 \qquad \dots \quad (5)$$

It may be noted that this equation differs from that given by Banerjee in the appearance of 3/2 in place of unity. As a result, one obtains the eigenvalue expression

$$W = m_0c^2\left[1+\frac{\alpha^2z^2}{\{(n_r+\frac{1}{2})+\sqrt{(l+2l'+1)^2-\alpha^2z^2}\}^2}\right]^{-\frac{1}{2}} \qquad \dots \quad (6)$$

which differs from the Sommerfeld's formula in the replacement of n_r by $(n_r+\frac{1}{2})$.

It is, therefore, concluded that the new degree of freedom, obtained by introducing the fifth dimension, does not produce the same effects as the electron spin.

REFERENCES

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